

Solutions to Maths workbook – 1 | Trigonometric Identities and Equations

Level - 2	Daily Tutorial Sheet - 10
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121.(C) $\tan\left(x + \frac{\pi}{4}\right) = 3 \tan 3x \Rightarrow \frac{\tan x + 1}{1 - \tan x} = 3 \left[\frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x} \right]$.

$\tan \alpha, \tan \beta, \tan \gamma, \tan \delta$ are roots of this equation.

$$\Rightarrow \tan \alpha + \tan \beta + \tan \gamma + \tan \delta = \frac{-\text{coeff. of } \tan^3 x}{\text{coeff. of } \tan^4 x} = \frac{0}{3} = 0$$

122.(B) $\sum \frac{\cos A}{\sin B \sin C} = \frac{\sum \sin A \cos A}{\sin A \sin B \sin C} = \frac{\sin 2A + \sin 2B + \sin 2C}{2 \sin A \sin B \sin C}$

For $A + B + C = \pi$ $\sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C \Rightarrow \frac{\sum \sin 2A}{2 \sin A \sin B \sin C} = 2$

123.(B) $P_n = \cos^n \theta + \sin^n \theta \Rightarrow P_n - P_{n-2} = \cos^n \theta + \sin^n \theta - \cos^{n-2} \theta - \sin^{n-2} \theta$
 $= \cos^{n-2} \theta (\cos^2 \theta - 1) + \sin^{n-2} \theta (\sin^2 \theta - 1) = \cos^{n-2} \theta (-\sin^2 \theta) + \sin^{n-2} \theta (-\cos^2 \theta)$
 $= -\sin^2 \theta \cos^2 \theta (\cos^{n-4} \theta + \sin^{n-4} \theta) = -\sin^2 \theta \cos^2 \theta P_{n-4}$

124.(B) $\tan^2 x + \sec x = a \Rightarrow \sec^2 x + \sec x - 1 = a \Rightarrow t^2 + t - 1 = a$

$$\Rightarrow t^2 + t + \frac{1}{4} = a + \frac{5}{4}$$

For $t \in (-\infty, -1] \cup [1, \infty)$

$$\left(t + \frac{1}{2}\right)^2 \text{ has min. value at } t = -1 \text{ i.e. } \left(t + \frac{1}{2}\right)^2 \in \left[\frac{1}{4}, \infty\right)$$

Hence, at least one solution iff

$$a + \frac{5}{4} \in \left[\frac{1}{4}, \infty\right)$$

$$a \in [-1, \infty)$$

125.(B) $\cot \alpha + \frac{1}{2} \tan \frac{\alpha}{2} = \frac{1 - \tan^2 \frac{\alpha}{2}}{2 \tan \frac{\alpha}{2}} + \frac{1}{2} \tan \frac{\alpha}{2} = \frac{1}{2 \tan \frac{\alpha}{2}} = \frac{1}{2} \cot \frac{\alpha}{2}$

Similarly, $\frac{1}{4} \tan \frac{\alpha}{4} + \frac{1}{2} \cot \frac{\alpha}{2} = \frac{\cot \frac{\alpha}{4}}{4} \therefore$ Equation reduces to $\frac{\cot \frac{\alpha}{2^n}}{2^n}$

126.(A) $\sin^3 x \sin 3x = \sin^3 x (3 \sin x - 4 \sin^3 x) = 3 \sin^4 x - 4 \sin^6 x = 3(1 - \cos^2 x)^2 - 4(1 - \cos^2 x)^3$
 $\Rightarrow 3(1 + t^4 - 2t^2) - 4(1 - t^6 - 3t^2 + 3t^4)$ (where $t = \cos x$), Now $C_0 + C_2 + C_4 + C_6 = 0$

127.(C) $2 - \cos x + 1 - \cos^2 x$, Let $\cos x = t \Rightarrow 3 - t^2 - t$

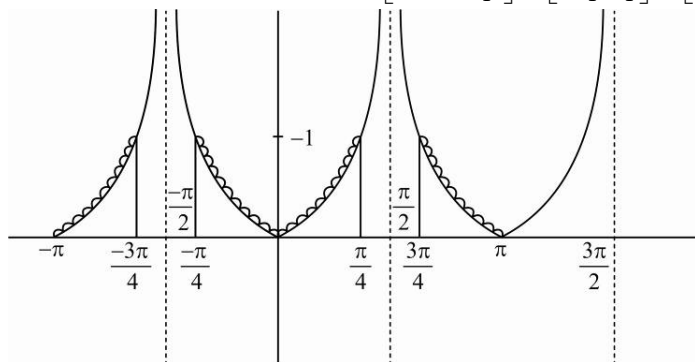
This is quadratic in t with $-1 \leq t \leq 1$. Now find range. $\left[\begin{array}{l} \text{greatest value} = 13/4 \\ \text{least value} = 1 \end{array} \right]$

128.(D) $\sin 5\theta \cos 3\theta = \sin 9\theta \cdot \cos 7\theta$

$$\frac{\sin 8\theta + \sin 2\theta}{2} = \frac{\sin 16\theta + \sin 2\theta}{2} \Rightarrow \sin 8\theta = \sin 16\theta, \quad \sin 8\theta = 2 \sin 8\theta \cos 8\theta$$

$$\Rightarrow \sin 8\theta = 0 \Rightarrow \theta = 0, \frac{\pi}{8}, \frac{2\pi}{8}, \frac{3\pi}{8}, \frac{4\pi}{8} \quad \text{or} \quad \cos 8\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{24}, \frac{5\pi}{24}, \frac{7\pi}{24}, \frac{11\pi}{24} \quad \therefore \quad 9 \text{ solutions}$$

129.(A) Graphically we can see that: $x \in \left[-\pi, -\frac{3\pi}{4}\right] \cup \left[-\frac{\pi}{4}, \frac{\pi}{4}\right] \cup \left[\frac{3\pi}{4}, \pi\right]$



130.(B) LHS = $(1 + \tan 1^\circ)(1 + \tan 44^\circ)(1 + \tan 2^\circ)(1 + \tan 43^\circ) \dots (1 + \tan 45^\circ)$

As, if $A + B = \frac{\pi}{4}$, then $(1 + \tan A)(1 + \tan B) = 2 \left\{ \tan \frac{\pi}{4} = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B} \right\}$

$$\Rightarrow (1 + \tan 1^\circ + \tan 44^\circ + \tan 1^\circ \tan 44^\circ) + (1 + \tan 2^\circ + \tan 43^\circ + \tan 2^\circ \tan 43^\circ) \dots (1 + \tan 45^\circ)$$

$$\Rightarrow (1+1)(1+1) \dots (1+1) 23 \text{ times} = 2^{23} \Rightarrow n = 23$$